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Two-dimensional heat and moisture transfer analysis of a cylindrical moist object subjected to drying: A finite-difference approach

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Abstract

In this paper a two-dimensional numerical analysis of heat and moisture transfer during drying of a cylindrical object is presented. Drying is a process of simultaneous heat and moisture transfer whereby moisture is vaporized by means of a drying fluid (e.g., air), as it passes over a moist object. The two-dimensional analysis of heat and moisture transfer during drying of a cylindrical object is carried out using an explicit finite-difference approach. Temperature and moisture distributions inside the moist objects are obtained for different time periods and the results predicted from the present analysis are compared with two sets of experimental data available in the literature. A considerably high agreement is found between the predicted and measured values.

1. Introduction

Drying is an energy intensive operation consuming 9–25% of national industrial energy in the developed countries [1]. It appears to be a complicated process involving simultaneous, coupled heat and mass transfer phenomena, which occur inside the object being dried. The temperature and moisture distributions inside the moist solids during drying are of great importance in the drying industry in order to provide better design and performance analysis of the drying process. Due to intricate coupling between the primary variables and in combination with non-linearities as a result of variation of physical properties, the use of numerical simulation has become an essential tool in this regard [2,3].

The principle of modeling is based on having a set of mathematical equations, which can adequately characterize the process. The solution of these equations must allow the prediction of the process and moisture transfer parameters as a function of time. Numerical modeling is considered to be cost effective and in combination with the current state-of-the-art drying knowledge, it enables us to advance the fundamental understanding of the overall process. Numerous analytical and computational models have been proposed in the literature by various researchers [4–12] to study heat and moisture transfer analysis during drying of various solid objects. In most of the numerical models (e.g., [6,7,13]), the researchers employed finite element control volume technique to study moisture distributions within the wet solids and compare their calculations with the experimental data available at certain drying conditions, while analytical models used simplified approaches for the solution, with constant moisture diffusivity.

The main goal of the present study is to simulate the drying process in an axi-symmetric cylindrical moist object by using the fundamental models of heat conduction and moisture diffusion and to develop a computer code, which can adequately predicts the

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Nomenclature

A	constant in Eq. (6)
B	constant in Eq. (6)
$C_1, \ldots, C_8$	coefficients
c_p	specific heat capacity at constant pressure (J/kg K)
D	moisture diffusivity (m^2/s)
D_0	pre-exponential factor of Arrhenius equation (m^2/s)
h	heat transfer coefficient (W/m^2 K)
h_m	moisture transfer coefficient (m/s)
k	thermal conductivity (W/mK)
L	height (m)
m	number of mesh points of the numerical grid in r-direction
M	moisture content (kg/kg, db)
n	number of mesh points of the numerical grid in z-direction
r	radial coordinate
R	radius (m)
RH	relative humidity (kg/kg, db)
t	time (s)
T	temperature (K)
z	axial coordinate
Greek symbols	
α	thermal diffusivity (m^2/s)
ρ	density (kg/m^3)
θ	dimensionless temperature
ϕ	dimensionless moisture content
Subscripts	
d	drying air
i	initial
m	moisture

temperature and moisture distributions inside the wet object with temperature dependent diffusivity and validate the predicted results with the experimental data obtained from literature.

2. Modeling

The mathematical models used to simulate the drying process are two-dimensional Fourier law of heat conduction and Fick's law of mass diffusion. The governing Fickian equation is exactly in the form of the Fourier equation of heat conduction, in which temperature and thermal diffusivity are replaced with moisture content and moisture diffusivity, respectively. In this regard, the assumptions considered in the simulation are as follows: (i) constant thermophysical properties, (ii) negligible shrinkage or deformation of object during drying, (iii) negligible heat generation inside the moist object, (iv) constant drying air temperature, (v) temperature-dependent moisture diffusivity, and (vi) two-dimensional variation of temperature and moisture in cylindrical object (i.e., r and z).

The mathematical equations governing the drying process in a two-dimensional cylindrical object with the appropriate boundary conditions are given as:

- Heat transfer:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} \tag{1}$$

with the following boundary conditions:

$T(r,z,0) = T_i$

$$\text{at } r = 0; \quad \frac{\partial T(0,z,t)}{\partial r} = 0$$

$$\text{at } r = R; \quad -k\frac{\partial T(R,z,t)}{\partial r} = h(T_d - T)$$

$$\text{at } z = 0; \quad -k\frac{\partial T(r,0,t)}{\partial z} = h(T_d - T)$$

$$\text{at } z = L; \quad -k\frac{\partial T(r,L,t)}{\partial z} = h(T_d - T)$$

- Moisture transfer:

$$\frac{1}{D}\frac{\partial M}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial M}{\partial r}\right) + \frac{\partial^2 M}{\partial z^2} \tag{2}$$

with the following boundary conditions:

$M(r,z,0) = M_i$

$$\text{at } r = 0; \quad \frac{\partial M(0,z,t)}{\partial r} = 0$$

$$\text{at } r = R; \quad -D\frac{\partial M(R,z,t)}{\partial r} = h_m(M_d - M)$$

$$\text{at } z = 0; \quad -D\frac{\partial M(r,0,t)}{\partial z} = h_m(M_d - M)$$

$$\text{at } z = L; \quad -D\frac{\partial M(r,L,t)}{\partial z} = h_m(M_d - M)$$

where D is the moisture diffusivity, whose dependence on temperature is of the form of Arrhenius equation [7]:

$$D = D_0 \exp\left(\frac{-1119}{T}\right) \tag{3}$$

The temperature in the heat conduction equation and moisture content in the diffusion equation are non-dimensionalized using the following equations

$$\theta = \frac{T - T_i}{T_d - T_i} \tag{4}$$

$$\phi = \frac{M - M_d}{M_i - M_d} \tag{5}$$

3. Solution methodology

The solution of the above governing equations is difficult to obtain using analytical methods. Moreover, considerable assumptions have to be considered in order to obtain a closed form solution with many inadequacies. Therefore, approximate methods of solution are used to solve them. The method used in the present study is the explicit finite difference approximation where the governing equations are first transformed into difference equations by dividing the domain of solution to a grid of points in the form of mesh and the derivatives are expressed along each mesh point referred to as a node. Numerical grid of an axi-symmetric cylindrical object is shown in Fig. 1. The index i represents the mesh points in the r-direction starting with $i = 0$ being one boundary and ending at $i = m$, the other boundary while index j represents the mesh points in the z-direction starting from $j = 0$. Thus, the finite difference representation of the mesh points will be as follows:

$$r_i = i\Delta r \quad \text{for } i = 0, 1, 2, \ldots, m$$
$$z_j = j\Delta z \quad \text{for } j = 0, 1, 2, \ldots, n$$

where Δr and Δz represents the grid sizes in the r- and z-directions respectively and the subscripts denote the location of the dependent variable under consideration,

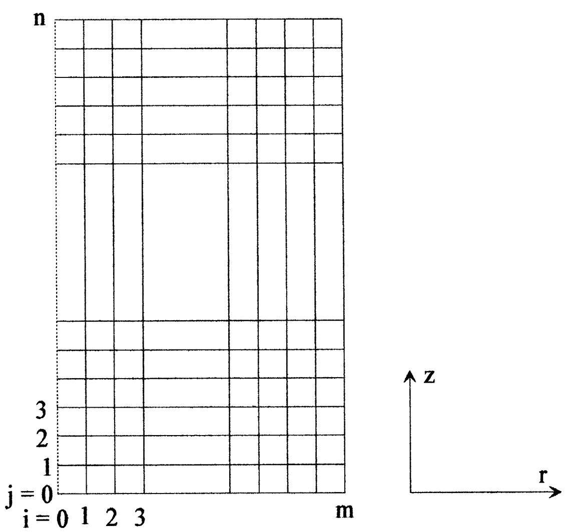

Fig. 1. Numerical grid for an axi-symmetric cylindrical moist object.

Table 1
Drying conditions and product properties used in the simulation and variation of the first experiment data set

Product	Cylindrically shaped broccoli
Size	Diameter 0.007 m and length 0.02 m
T_i	298 K
T_d	333 K
M_i	9.57 g/g (db)
RH	1.18 g.g (db)
k	$0.148 + 0.493 \times M_i$ (W/mK)
ρ	2195.27 kg/m^3
c_p	$(0.837 + 1.256 \times M_i) \times 1000$ (J/kg K)
Reference	Simal et al. [14]

i.e., $T_{i,j}$ means the temperature at the ith r-location and jth z-location. Knowing the value of dependent variable at the initial time step, unknown values at next time steps are calculated using the finite difference equations. The finite difference representations of the governing equations can be written in the following form:

- Heat transfer

$$T_{i,j,k+1} = AT_{i+1,j,k} + (1 - 2[A + B])T_{i,j,k} + AT_{i-1,j,k} + B(1 + 0.5j)T_{i,j+1,k} + B(1 - 0.5j)T_{i,j-1,k} \tag{6}$$

where $A = (\alpha\Delta t)/(\Delta z)^2$ and $B = (\alpha\Delta t)/(\Delta r)^2$.

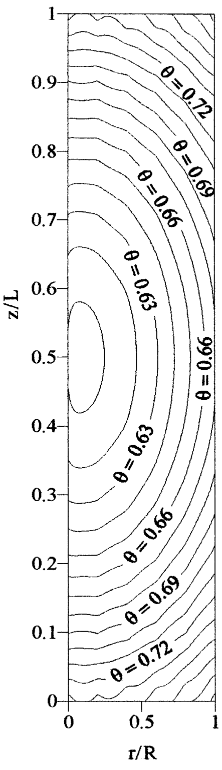

Fig. 2. Dimensionless temperature distribution inside an axi-symmetric cylindrical object at 180 s.

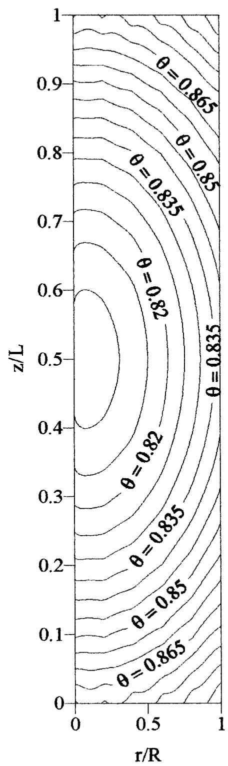

Fig. 3. Dimensionless temperature distribution inside an axisymmetric cylindrical object at 300 s.

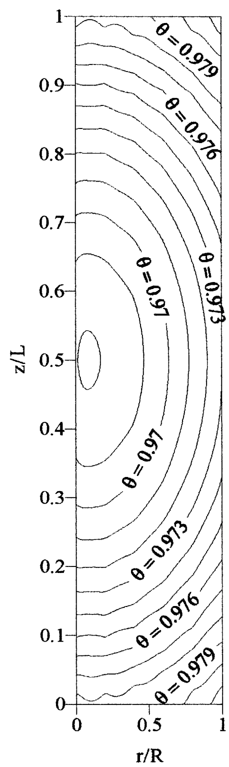

Fig. 4. Dimensionless temperature distribution inside an axisymmetric cylindrical object at 600 s.

The initial and boundary conditions are:

$$T_{i,j,0} = T_{\mathrm{i}}$$

$$\text{at } r = 0; \quad T_{0,j,k} = T_{1,j,k}$$

$$\text{at } r = R; \quad T_{m,j,k} = \frac{T_{m-1,j,k} + C_5 \times T_{\mathrm{d}}}{1 + C_5}$$

$$\text{at } z = 0; \quad T_{i,0,k} = \frac{T_{i,1,k} + C_6 \times T_{\mathrm{d}}}{1 + C_6}$$

$$\text{at } z = L; \quad T_{i,n,k} = \frac{T_{i,n-1,k} + C_6 \times T_{\mathrm{d}}}{1 + C_6}$$

where $C_5 = (h\Delta r)/k$; $C_6 = (h\Delta z)/k$.

- Moisture transfer

$$M_{i,j,k+1} = A_m M_{i+1,j,k} + (1 - 2[A_m + B_m])M_{i,j,k} + A_m M_{i-1,j,k} + B_m(1 + 0.5j)M_{i,j+1,k} + B_m(1 - 0.5j)M_{i,j-1,k} \quad (7)$$

where $A_m = (D\Delta t)/(\Delta z)^2$ and $B_m = (D\Delta t)/(\Delta r)^2$.

The initial and boundary conditions are:

$$M_{i,j,0} = M_{\mathrm{i}}$$

$$\text{at } r = 0; \quad M_{0,j,k} = M_{1,j,k}$$

$$\text{at } r = R; \quad M_{m,j,k} = \frac{M_{m-1,j,k} + C_7 \times M_{\mathrm{d}}}{1 + C_7}$$

$$\text{at } z = 0; \quad M_{i,0,k} = \frac{M_{i,1,k} + C_8 \times M_{\mathrm{d}}}{1 + C_8}$$

$$\text{at } z = L; \quad M_{i,n,k} = \frac{M_{i,n-1,k} + C_8 \times M_{\mathrm{d}}}{1 + C_8}$$

where $C_7 = (h_{\mathrm{m}}\Delta r)/D$; $C_8 = (h_{\mathrm{m}}\Delta z)/D$.

The above difference equations are used to obtain temperature and moisture distributions inside the cylindrical object at different time periods. The grid independent tests are conducted to ensure the grid independent results in the simulation. Stability analysis is performed in order to investigate the boundedness of the exact solution of the finite-difference equations using the von Neumann's method. The method introduces an initial line of errors as represented by a finite-Fourier series and applies in a theoretical sense to initial value problem. The stability criterions obtained for the above difference equations are:

$$\Delta t \leqslant \frac{(\Delta r)^2(\Delta z)^2}{2\alpha[(\Delta r)^2 + (\Delta z)^2]} \quad \text{for heat transfer} \quad (8)$$

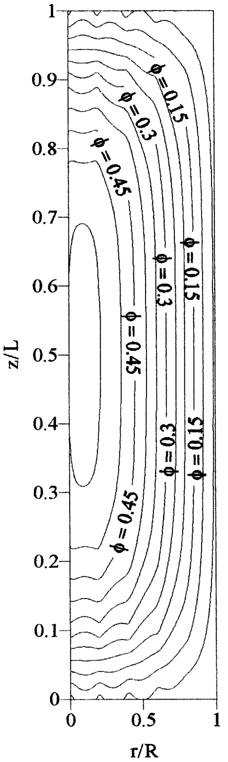

Fig. 5. Dimensionless moisture distribution inside an axi-symmetric cylindrical object at 180 s.

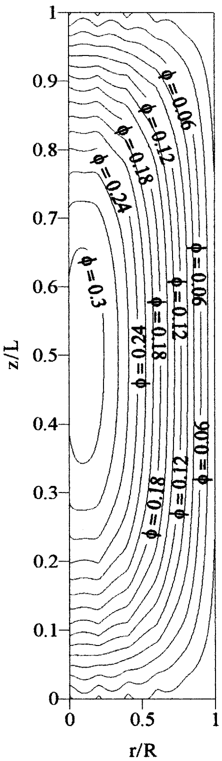

Fig. 6. Dimensionless moisture distribution inside an axi-symmetric cylindrical object at 300 s.

$$\Delta t \leqslant \frac{(\Delta r)^2(\Delta z)^2}{2D[(\Delta r)^2 + (\Delta z)^2]} \quad \text{for moisture transfer} \tag{9}$$

Therefore, the above conditions are expected to be satisfied in order to have converged solution.

4. Results and discussion

After presenting the analysis part, here we undertake a simulation study for heat and moisture transfer during drying of a cylindrical moist product to predict the temperature and moisture profiles axi-symmetrically at different drying periods. The product considered in the simulation was a cylindrically cut broccoli with a diameter of 0.007 m and a length of 0.02 m. The thermophysical properties and the experimental drying conditions for the object considered in the simulation are listed in Table 1.

The dimensionless temperature distributions inside the axi-symmetric cylindrical moist object for different drying times are shown in Figs. 2–4. As seen in the figures, the temperature inside the moist object increases as the drying time progresses, which is due to the higher drying air temperature than the temperature of the object. Moreover, temperature distributions inside the moist object appear to be non-uniform and time-dependent, and this clearly gives an indication that the temperature dependent moisture diffusivity varies inside the object, which in turn affects the rate of moisture diffusion in the object.

In addition, the dimensionless moisture distributions inside an axi-symmetric cylindrical object for different drying periods are shown in Figs. 5–7. As the figures show, the moisture content inside the moist object reduces as the time period increases as a result of drying process. The reduction rate of moisture content in the surface region is higher as compared to the interior of the object. Moreover, in the early drying period moisture content reduces rapidly and as the drying period progresses the rate of reduction of moisture content becomes almost steady. This is more pronounced at the surface vicinity. The rapid drop of moisture content in the early drying period is because of the high moisture gradient in this region, which in turn derives considerable diffusion rates from inside to the surface.

In the second part of the study we go one step ahead and verify the model. In this regard, the temperature and moisture distributions predicted for two different cylindrical moist products using the numerical simulation are compared with two sets of experimental data taken from the literature (for a cylindrically shaped broccoli from

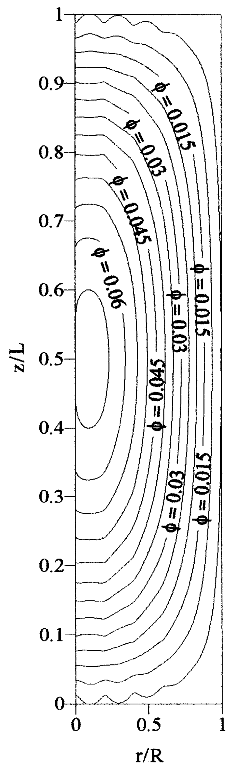

Fig. 7. Dimensionless moisture distribution inside an axi-symmetric cylindrical object at 600 s.

Table 2
Drying conditions and product properties of the second experiment set used for validation

Product	Cylindrically shaped apple
Size	Diameter 0.0037 m and length 0.008 m
T_i	298 K
T_d	333 K
M_i	5.689 g/g (db)
RH	2 g/g (db)
k	0.567 (W/mK)
ρ	856 kg/m^3
c_p	3683.65 (J/kg K)
Reference	Feng et al. [15]

Simal et al. [14] and for a cylindrically shaped apple from Feng et al. [15]). The experimental drying conditions, geometrical details and thermophysical properties of the moist objects as taken from Ref. [16] are listed in Tables 1 and 2. The comparisons between the predicted temperature and moisture distributions with the first set of experimental data are shown in Figs. 8 and 9. Due to non-availability of temperature distribution inside the object for the second set of experimental data, only a comparison of predicted and experimental moisture distributions is shown in Fig. 10. As seen from the figures, a considerably high agreement is found between the predicted results and the measured values taken from literature. Consequently, this shows that the numerical methodology presented above with a developed computer code is capable of modeling heat and moisture transfer inside the cylindrical moist objects during drying.

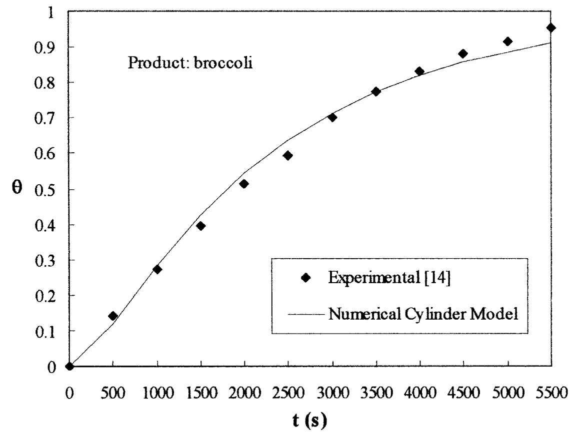

Fig. 8. Comparison between predicted and calculated center temperature distributions in a cylindrical moist object for the first experimental data set.

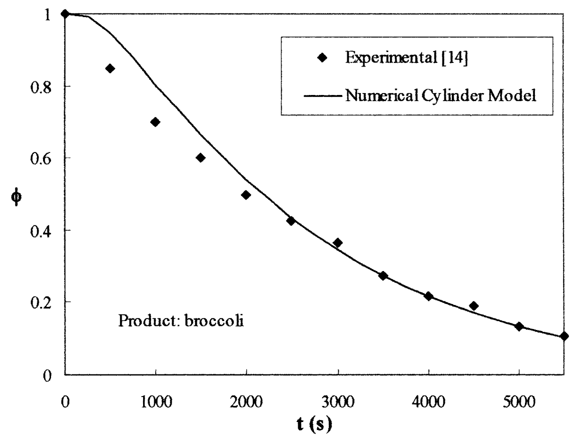

Fig. 9. Comparison between predicted and calculated center moisture distributions in a cylindrical moist object for the first experimental data set.

5. Conclusions

The numerical solution for the temperature and moisture distributions inside an axi-symmetric cylindrical object subjected to drying is presented. It is found that the temperature in the early drying period rises rapidly and as the drying period progresses, the rise of

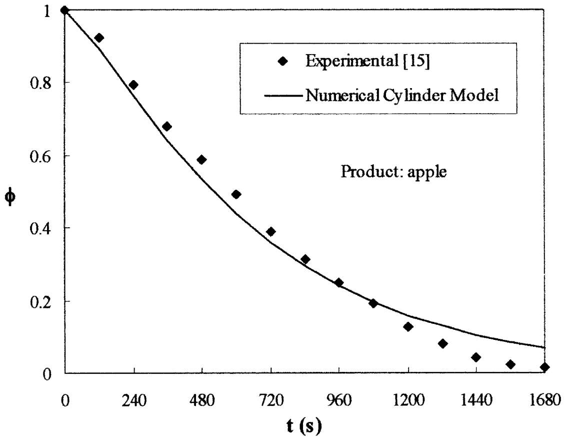

Fig. 10. Comparison between predicted and calculated center moisture distributions in a cylindrical moist object for the second experimental data set.

temperature attains almost steady. The moisture gradient is higher in the early drying period and as drying progresses, the moisture gradient remains almost steady. Moreover, validation of the results obtained from the present study is performed with two sets of experimental data available in the literature. A considerably high agreement is found between the predicted and measured values for the temperature and moisture distributions inside the object.

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